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Analysis of Propagating Explosions

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Abstract

Weapons are often in close proximity to one another during transport or storage. If one weapon explodes, there is a possibility that the fragments generated will initiate a subsequent explosion in one or more neighboring weapons. Propagating explosions of this sort have the potential for severe consequences either because of the total amount of explosives that react or because the response of individual weapons may be particularly energetic. In this paper, we consider a well-defined problem in which the nature of the progression to all possible end states can be studied. We wish to determine the expected number of weapons to detonate along with other useful quantities. We examine the possible end states that the system can reach and show that we can represent the propagation process as a series of discrete time transitions. The transition probabilities from one state to the next then will depend only on the present state of the system. We present results of simulations that illustrate the effect of varying the detonation probability parameters .

Introduction

Weapons containing significant quantities of high explosives (HE) are sometimes located in close proximity to one another. If an explosion occurs in a weapon, the possibility of propagation to one or more additional weapons may exist, with severe consequences possibly resulting. In the general case, a system of concern consists of multiple weapons and various other objects in a complex, three-dimensional geometry. Analysis of this problem requires an approach that can both define the circumstances under which rare events can occur and calculate the probability of such occurrences. We have developed such an approach based on combining process-tree methodology with Monte Carlo transport simulation and described it elsewhere (Ref. 1). In this work, we extend these ideas to enable the investigation of problems created by the possibility of realizing certain low-probability, chain-reaction sequences of explosions. Such sequences may contain individual undesired events made possible by the special environment created during the explosion sequence or simply by the collective output of a long explosion chain.

Our approach is based on characterizing the accident progression by using damage-state vectors. These vectors describe the system state completely, including path information, and are equally useful for both temporary states and the final end states. We investigated a problem involving a collection of re-entry vehicles (RVs) surrounding a solid rocket motor (SRM). This problem has the unusual feature of a terminating mechanism (the possibility that the SRM could explode, thus artificially ending any propagating chains in progress) and the fact that no state of the system can be entered more than once.

In this paper, we define this problem further, explain our technique for mathematically characterizing the accident progression by means of damage-state vectors, and investigate the mathematical properties of Markov techniques that apply to the problem at hand. Several features of our problem are unusual, so much of the extensive body of Markov-related work available in the literature does not apply directly. We outline the numerical simulation model developed and show results for an interesting set of problem parameters. Prospects for extension of the approach and generalization of application problems are considered.

Scenario

The model geometry consists of a ring of n RVs surrounding a solid fuel rocket motor as shown in Figure 1 for the case $n = 8$. One of the RVs, referred to here as the *donor*, is assumed to detonate and is the original source for the fragments of concern. Other RVs are in the line of sight of the donor. We refer to the two closest units as *adjacent acceptors (AA)*. It is assumed that fragments from the donor are distributed equally about a line of symmetry drawn through the centers of the donor and the SRM. Therefore, the left and right acceptors see the same fragment flux. In the general case, it is also possible for fragments to reach additional RVs. Here we consider that only the next two RVs beyond the AAs are in the direct fragment path from the donor. We denote these units as *next adjacent acceptors (NAAs)*.

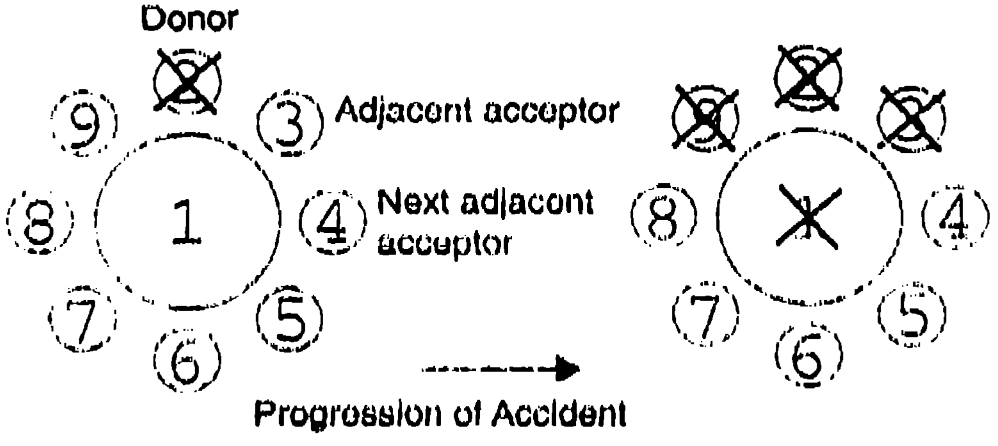

Figure 1 – Basic accident model showing one possible propagation scenario.

Fragments from the donor can cause a subsequent detonation in *AAs* or *NAAs* or in the *SRM*. This makes a sequence of RV detonations possible, a chain reaction. To make the problem more tractable, we have made two simplifying assumptions: the chain reaction can be represented as a set of discrete generations at uniform intervals and, because the RVs are identical, detonation of any acceptor acts as a new donor in the next generation. In the problem examined here, it is assumed that detonation of the SRM terminates the sequence. That is, only acceptors that are impacted by fragments from the original or subsequent donors before or simultaneously with SRM detonation can detonate as the result of an RV fragment.

The evolution of two potential chain reactions is shown in Figure 2. In the first generation, the original donor (Object 2) detonates. In the upper chain, fragments from the donor lead to daughter detonations in the AAs denoted by Object 3 and 9. Fragments from these detonations in turn produce detonations in their own AAs—Objects 4 and 8—as well as in the SRM, Object 1. This terminates the sequence with a total of four RVs detonated. In the lower chain, the original donor causes a detonation in an NAA, Object 4. In the third generation, this daughter donor causes both of its AAs as well as the SRM to detonate, again ending the sequence. It can be seen that many other detonation chains are possible, and we consider next a general methodology for enumerating the complete set of end states and the probability associated with each.

Theory

The evolution of a chain of RV detonations depends on three probabilities. These are the probability of detonation for an adjacent acceptor, p_{AA}; for a next adjacent acceptor, p_{NAA}; and for the SRM, p_{SRM}. In general, these probabilities depend on the fragment generation process, the transport of the fragments to the acceptors and SRM, and the response of the energetic material (HE or propellant) in the target. The development of a set of models to calculate the component probabilities is beyond the scope of this paper and is discussed elsewhere (Ref. 1). Here we treat the three probabilities as parameters and evaluate the response of the system accordingly.

If p_{SRM} is less than 1.0, a chain of RV detonations becomes possible. Various end states, that is, patterns of exploded and unexploded RVs, exist. These end states depend on the relative magnitudes of p_{AA} and p_{NAA}. Any particular end state may be reached by multiple paths. For example, explosions in Objects 3 and 4 may occur directly in generation 2 as a result of fragments from the original donor, or Object 4 may be detonated in generation 3 by a fragment from Object 3. Thus, to calculate the total probability of detonating i RVs, $1 \leq i \leq n$, we must calculate the probability of occurrence for all possible paths.

Even for small n, a large number of formal probabilities exist, and it is necessary to use some symbolic scheme to keep track of the paths. We have chosen to use an n-tuple identification sequence to track the system damage vector. This tracking is required to

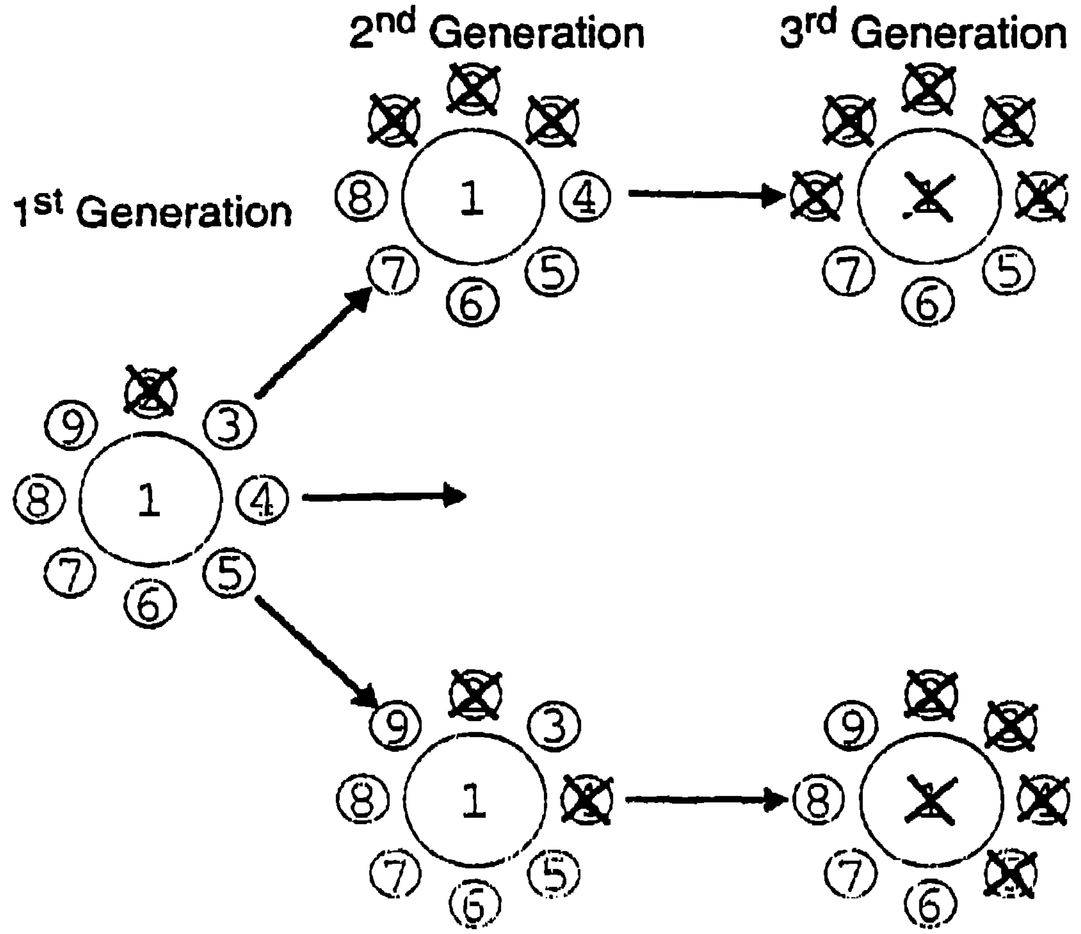

Figure 2 - Basic accident model showing possible propagation scenarios.

calculate the transition probabilities between damage states. The scheme is shown in Figure 3 for n = 8. Each RV has a unique position in the octuple that corresponds to its numerical identification; the SRM has the lead position in the octuple. An object is represented by a zero if it is undetonated. If an object detonates in some generation, the zero is replaced by the number of the generation in which the explosion in that object occurred. For example, all chains begin in the first generation as [010000000] with the detonation of the original donor. A particular state in the second generation might be the detonation of a single NAA, which would be designated as [010200000]. Then perhaps in the third generation, the SRM, as well as both the adjacent acceptors for Object 4, explode. This state, which is an end state because of the SRM explosion, would be [313230000]

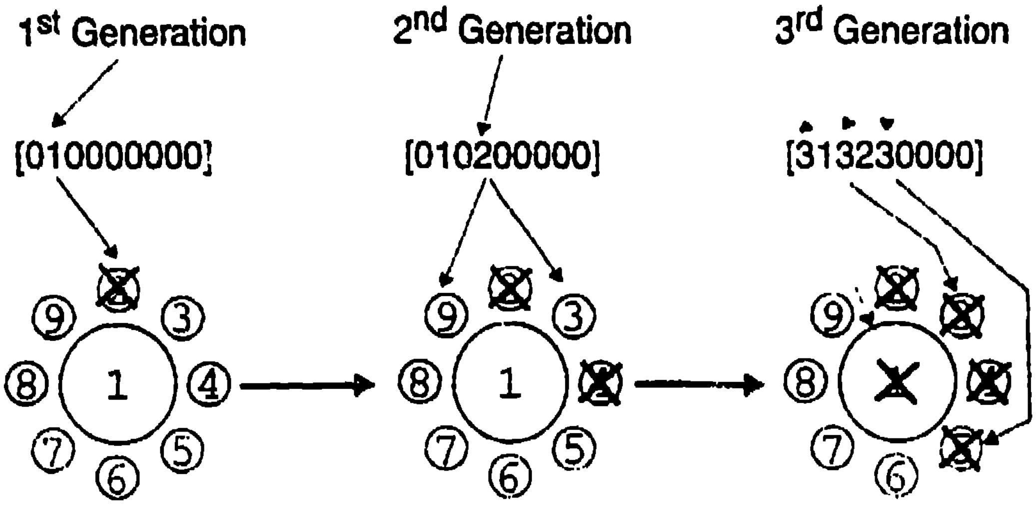

Figure 3 – Identification of damage-state vector for multiple generations of propagation scenarios.

It is important to note that for any formally possible n-tuple of generation j, n_j, we can list the transitions to all possible damage-state vectors in generation j+1. Further, because we know the transition probabilities associated with the three fundamental events, detonation of AAs, NAAs and the SRM, we can calculate the probability for each possible state in generation j+1 given the value of n_j. The fact that the transition from a state in generation j s_j to some state in generation j+1, s_{j+1} only depends on the value of n_j and no previous state means that the detonation sequence is a Markov chain. We discuss the implications of this in the next section.

Markov Chains

The model of propagation generated in this analysis is a Markov chain with nonstationary transition probabilities (Ref. 2). The model is a Markov chain because the conditional probability p_{ij} that state i is reached from state j is independent of the path to state j. This is stated mathematically by

$$p(X_{i+1} | X_i, X_{i-1}, \ldots, X_1) = p_{ij}$$

The state space for the chain is the set of different combinations of exploded and unexploded RVs in combination with the SRM state. Each state has at most 32 accessible states associated with it because each exploding RV can be the donor for only four other RVs and the SRM. Transitions in the Markov chain represent changes between states caused by fragments from an exploded RV striking an unexploded RV or the SRM. All explosions and fragment transport are assumed to take place within one generation.

The transition probabilities are nonstationary because they depend on the number of generations since explosion of an RV. In the basic model, no state can transition to another state after it has existed for more than one generation. This represents the case where fragments generated by an explosion in one generation do not cause an explosion in any other object. Thus, the p_{ij} for a state has a nonzero value only in the generation immediately following a transition to that state.

In an extension of the model that allows for the treatment of deflagration-to-detonation transition, an incubation period of a set number of generations is allowed before an object explodes. This extension allows for the possibility of detonation delay in an object. With this extension, transitions may occur during a set number of generations after an object has exploded. The chain is still Markov even with this extension because the transition probabilities remain path-independent.

This Markov chain has no communicating states. Each state has a set of accessible states, but a state cannot be entered more than once, so the chain is a special form of a branching chain. Another unusual feature of this chain is the terminating feature of the SRM explosion. No state transitions occur after an SRM explosion, so all the states with SRM explosion (a non-zero value in the leading position of the state vector) are terminal states.

The advantage to using the Markov chain model is realized in the probability calculations. A large number of possible states may be enumerated, and the probability of each may be calculated exactly using this chain model. The chain model is, of course, an idealization of the actual situation, but many insights may be gained into the process using this relatively simple model. It provides a straightforward way of illustrating the proliferation of possible states and shows the effects of changing relative probabilities among the communicating elements of the model.

Simulation Model

The simulation model is based on a simple approach. There is only one beginning state. A donor is assumed to explode; the results are independent of which of the RVs is chosen. The accident simulation proceeds as the accident does, that is, generation by generation. Often different outcomes are possible, for example, an acceptor may explode as a result of fragment impacts or it may not. Each generation may have a number of possible outcomes. Each possible outcome is recorded as a distinct possible damage state and then used as a starting point for the next generation calculation; eventually all realizable end states are identified.

This approach can be implemented with either a recursive or a nonrecursive algorithm. In both cases, a program unit to advance the accident state one generation is needed. This unit must examine a damage vector to determine if it represents an end state. If not, it identifies all the possibilities for the next generation and their associated probabilities. If the algorithm is recursive it simply "calls itself" for each of the damage-state possibilities for the new generation. If the algorithm is nonrecursive, all the new damage-state possibilities are added to a stack. The current state is always retained as one possible end state if one of the possible outcomes is that nothing happens. The recursive algorithm is finished when there are no additional damage states possible. The nonrecursive algorithm is finished when the stack is exhausted. Because different paths can lead to the same end state, post-processing is necessary to condense duplicate end states. In addition, it is convenient to collapse symmetric states; for example, [212000000] and [210000002] are considered equivalent, so their probabilities are summed and both states are represented as [212000000].

It is necessary to identify each possible next state as well as its probability to advance the accident state one generation. To obtain the possible next states, each unexploded acceptor is considered to determine if it can be an AA or NAA for some donor. The total numbers of donors for which it is an AA or an NAA are recorded separately for later use in computing probabilities. An example of this process is shown in Figure 4. All the possible next states are generated by looping over the permutations of possibilities for each acceptor to be an AA or NAA.

The probabilities corresponding to each of the possible next states are computed from factors multiplying the current state probability. There will be a multiplying factor for each possible acceptor. This factor is divided further into contributions from donors to AA positions and a contribution from donors to NAA positions. Although it is conceptually simple, formulation of the next state probabilities is complicated and tedious because every possibility for each potential acceptor must be considered. However, it is easy to illustrate the basic concepts for a simple case.

Consider the probabilities for the transition from generation 2 to generation 3 in Figure 4. Only four next states are possible because only one acceptor and the SRM remain

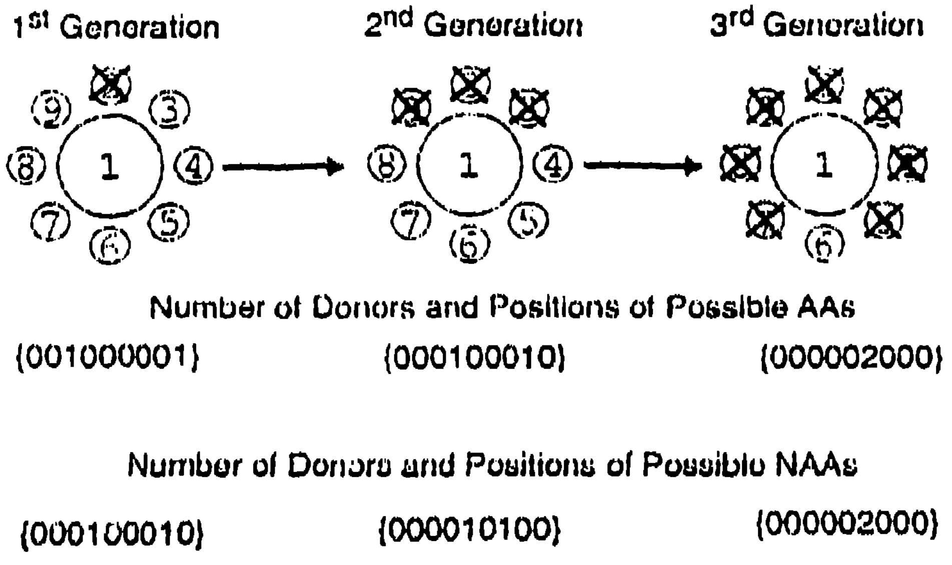

Figure 4 – Simplified illustration of the transition probability calculation

unexploded. To further simplify the illustration, let us ignore the complexity introduced by the SRM; then only two next states are possible. Those correspond to object 6 exploding or not exploding; that is, the possible end states are [012334332] and [012330332]. The *conditional* probability of Object 6 not exploding is given by

$$p_s = (1 - p_{AA})^2 (1 - p_{NAA})^2 .$$

Therefore, the probability of end state [012330332], where Object 6 does not detonate, is given by the probability of the current state times p_s. The probability of the remaining end state, [012334332], is given by 1—the previous result.

<u>Simulation Results</u>

The simulation model will work for any values of p_{AA}, p_{NAA}, and p_{SRM}. Illustrative results are discussed below for n = 8 with arbitrary and large detonation probabilities; in this case, p_{AA} = 0.95, p_{NAA} = 0.90, and p_{SRM} = 0.95. Table 1 shows the 20 highest probability end states obtained. The total number of distinct end states possible is quite large; in this case, it is 10,960.

Table 1 – Highest Probability End States for the Illustration Problem

Index	End state	Probability	Cumulative Probability
1	212000002	0.857375	0.857375
2	212000000	0.090250	0.947625
3	312230322	0.032987	0.980612
4	312230032	0.007330	0.987942
5	312230022	0.003472	0.991414
6	312230323	0.003298	0.994713
7	210000000	0.002375	0.997088
8	312300032	0.000406	0.997494
9	312230000	0.000405	0.997899
10	312200032	0.000386	0.998285
11	312230002	0.000386	0.998670
12	312300323	0.000366	0.999036
13	312230023	0.000174	0.999210
14	312200323	0.000174	0.999383
15	312230320	0.000174	0.999557
16	312200022	0.000091	0.999648
17	313230323	0.000082	0.999730
18	312300000	0.000043	0.999773
19	312300002	0.000043	0.999816
20	312200000	0.000021	0.999837

We chose this example because we wanted to know the likelihood of a long chain reaction of RV detonations in scenarios where the probability that the SRM would explode and terminate the chain is relatively high. The most likely end state is detonation of the two AAs and the SRM. This has a probability p = 0.85. However, the third most probable end state involves the detonation of seven RVs as well as the SRM. Detonation of only the original donor and the SRM alone is only the seventh most likely state in spite of the high probability of SRM detonation. It is interesting to note that the probability of realizing some long chain reactions is around a few per cent, which was considered to be rather large for the purposes of our study. To investigate this feature further, we collapsed the end states to summarize the results in terms of the total number of RVs detonated as shown in Table 2. The end states that are symmetric, i = 3, 5, 7 RVs detonated, are more likely than states just preceding them, i = 2, 4, 6. Note also that there is a large drop after i = 3 with the next most likely value being i = 7. These features arise from the fact that both p_{aa} and p_{naa} are large, which makes the transitions from i = 3 to i = 4, 5, 6 relatively unlikely to occur. The increase in detonation probability from i = 4 to i = 6 reflects the relative difficulty of these transitions. Detonation of all eight RVs is always a low-probability event.

To continue our exploration along these lines, we next investigated how the expected number of RVs detonated varies with p_{SRM}. These results are plotted in Figure 5; note that the values of p_{AA} and p_{NAA} were changed slightly from the preceding examples, as shown in the plot. It can be seen that even for relatively large values of p_{srm}, the expected number of RVs detonated is appreciably larger than three.

Conclusions

The Markov chain model described here is an effective tool for exploring the behavior of propagating explosions. It provides a convenient structure to make the problem tractable and to simplify the numerical effort associated with obtaining an exact solution for end

Table 2 – Summary Probabilities for the Illustration Problem

Number of Detonated RVs (including the first donor)	Probability
1	0.0024
2	0.0903
3	0.8574
4	0.0005
5	0.0014
6	0.0117
7	0.0364
8	0.0000

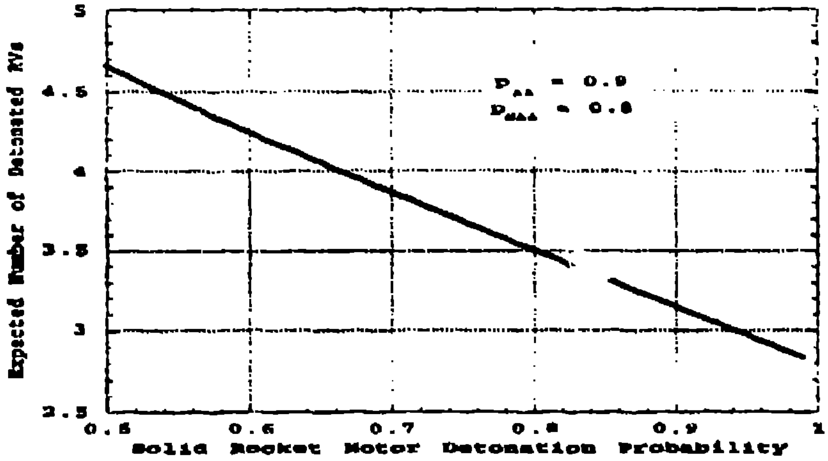

Figure 5 – Expected number of detonated RVs as a function of p_{SRM} for a slightly modified illustration problem.

states with low probabilities of occurrence. One of the insights gained from the chain model is the mere existence of so many states from such a simple problem. Some of the numerical results obtained were interesting and somewhat surprising. These include the high likelihood for the end state with seven RVs detonated and the increase in likelihood between four and seven detonations. The relatively weak power of SRM detonation probability in limiting the chain reaction is also of practical significance. In the future we plan to continue development of the model. Specific issues yet to be examined are the effect of delayed detonations, the effect of changing the number of RVs, and a more detailed study of the relationship between the component detonation probabilities.

References

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2. L. Breiman, Probability of Stochastic Processes, Boston, Houghton and Mifflin, 1969.